MODIFIED ROBUST RATIO-REGRESSION ESTIMATORS FOR POPULATION MEAN WITH AUXILIARY VARIABLE

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ABSTRACT
In sample survey, the uses of auxiliary variable in estimating population mean has been paramount for many researchers, because, it improves the efficiency of proposed estimators significantly. Several studies show that higher level of precision could be obtained when the auxiliary information were adopted especially when both variables were highly correlated. In this study, regression approach and sample size of auxiliary variable were used to improve the precision of proposed estimators. The bias and mean square error of each proposed modified ratio estimators using regression approach were obtained up to first order approximation. A theoretical comparisons of propose modified ratio estimators were carried out with other existing estimators for population mean. A numerical study was also carried out to see the performance and efficiency of the proposed estimators over some other existing estimators using Murthy (1967) and Mokhopadhyan (2009) data set. And the condition under which the proposed modified ratio estimators with the regression approach better than other existing modified ratio estimators were verified. Eventually, the study exhibits that proposed modified ratio estimators using regression approach were better than other existing modified ratio estimators as the proposed estimators has lower mean square error compared to other existing estimators.

Keywords: Ratio estimator, Regression Approach, Sample Size, Bias, Mean Square Error, Efficiency.

1. Introduction

The use of auxiliary information to increase or improve the efficiency of suggested or modified estimator in sampling theory has been a custom to many researchers in sample survey. In fact it more efficient when there are higher level of positive correlation between a studying variable and auxiliary. Auxiliary information incorporated with study variable contribute immensely in the estimation of population mean in finite population if its use properly. However, research exhibits that the uses of auxiliary variable incorporated with the study variable to modify ratio type estimator perform efficiently even when there existing negative correlation between the two variables. Many researchers have proposed an estimators by transformed conventional estimators such as ratio, product, different and regression estimators to relative type with adoption of auxiliary information in order to increase the precision of the estimate of the population mean.

The sample mean is the most acceptable and suitable estimator to estimate population mean when the sample size is large but research shows that regression approach play significance roles in efficiency of any estimator. Cochran W. G (1940) and Cochran WG (1977) initiated the use of auxiliary information and proposed a ratio estimator for population mean. It is so far established fact that the ratio type estimator provides better efficiency in comparison to simple mean estimator provided the study and auxiliary variables are positively correlated. If the correlation between the study and auxiliary variables is negative, product estimator is more efficient than sample mean estimator. Modified ratio estimators came into existence and were constructed by using one or more unknown constants. In a class of estimators, the estimator with minimum variance or mean square error is regarded as the most efficient estimator. This concept has been utilized by several researchers to improve the efficiency of ratio and product type estimators for estimating population mean of study variable like Upadhyaya L.N and Singh HP (1999), Singh H. O., Tailor R.,Kakran M. S. (2004), Jeelani M. I., Magbool S., Mir S.A (2013), Yadav S. K., Shukla A.K (2014), Abid M, Abbas N, Sherwani RAK and Nazir HZ (2016),Yadav S.K and Pandey H(2011), ,Yadav S.K and AdewaraA.A (2013), Yadav S.K, Mishra S.S and Shula A.K (2014), Yadav S.K, Mishra S.S and Shula A.K (2015), Yadav S.K, Mishra S.S, Shula A.K, Kumar S and Singh R.S (2016a), Yadav S.K, Gupta S.A.T, Mishra S.S, and Shula A.K

(2016b), Yadav S.K, Subramani J, Mishra S.S, and Shula A.K (2016c), Yadav S.K, Misra S, Mishra S.S, and Chutiman(2016d)

Let U= {U_1, U_2, U_3,..........,U_N }be a finite population having units and each U_i,=(X_i,Yi), i = 1, 2, 3,…,N has a pair of values. Y is the study variable and X is the auxiliary variable which is correlated with Y. Let y= $\{y_1, y_2, y_3, \ldots\ldots y_n\}$ and x = $\{x_1, x_2, x_3$ ….x_n} be n sample values, $\bar{y}$ and $\bar{x}$ are the sample means of the study and auxiliary variables respectively. Let S_Y^2 and S_X^2 be the finite population variance of Y and X respectively and S_y^2 and S_x^2 be respective sample variances based on the random sample of size n drawn without replacement. The following are the other notations used throughout this study.

N: Population size, n: Sample size,

Y: Study variable, X: Auxiliary variable

$\bar{y}, \bar{x}$: Sample means of study and auxiliary variables.

$\bar{Y}, \bar{X}$: Population means of study and auxiliary variables.

ρ :Coefficient of correlation of study and auxiliary variables

C_y , C_x: Coefficient of variations of study and auxiliary variables

β_1: Coefficient of skweness of auxiliary variable.

, β_2: Coefficient of kurtosis of auxiliary variable,

M_d : Median of the auxiliary variable

$$S_y^2 = \frac{1}{n-1}\sum_{i=0}^{n}(y_i - \bar{y})^2, \quad S_x^2 = \frac{1}{n-1}\sum_{i=0}^{n}(x_i - \bar{x})^2, \quad S_Y^2 = \frac{1}{N-1}\sum_{i=0}^{N}(Y_i - \bar{Y})^2,$$

$$S_X^2 = \frac{1}{N-1}\sum_{i=0}^{n}(X_i - \bar{X})^2, \quad \theta = \left(\frac{1}{n} - \frac{1}{N}\right), C_y = \frac{S_y}{\bar{Y}^2}, C_x = \frac{S_x}{\bar{X}^2}.$$

1. Literature review

The most suitable estimator for estimating population mean $\bar{Y}$ is the sample $\bar{y}$ given by

$$\bar{y} = \frac{1}{n}\sum_{i=0}^{n} y_i \qquad (1)$$

And it is unbiased for population mean and its variance up to first order of approximation this is given by,

$$V(\bar{y}) = \theta S_y^2 = \theta\bar{Y}^2 C_y^2 , \qquad (2)$$

Cochran (1940) proposed an estimator by using positive correlation of auxiliary variable with study variable and the following usual ratio estimator of population mean as,

$$\hat{\bar{Y}} = \bar{y}.\frac{\bar{X}}{\bar{x}} \qquad (3)$$

The above estimator is biased estimator of population mean and its Bias and Mean Square Error were obtained up to first order approximation respectively.

$$\text{Bias}(\hat{Y}_r) = \theta\bar{Y}\left(C_x^2 - \rho C_x C_y\right) \qquad (4)$$

$$\text{MSE}(\hat{Y}_r) = \theta\bar{Y}^2\left(C_x^2 + C_y^2 - 2\rho C_x C_y\right) \qquad (5)$$

Sisodia B. V. S. and Dwivedi V.K (1981) proposed ratio type estimator by modified auxiliary information with the uses of coefficient of variation (C_x) of auxiliary variable as:

$$\hat{\bar{Y}}_1 = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right) \qquad (6)$$

$$\text{Bias}(\hat{\bar{Y}}_1) = \theta\bar{Y}\left(\delta_1^2 C_x^2 - 2\rho C_x C_y\right) \qquad (7)$$

$$\text{MSE}(\hat{\bar{Y}}_1) = \theta\bar{Y}^2\left(C_y^2 + \delta_1^2 C_x^2 + -2\rho C_x C_y\right), \qquad (8)$$

where it's constant is given as: $\delta_1 = \frac{\bar{X}}{\bar{X}+C_x}$ (9)

Upadhyaya L. N., Singlh H. P,(1999) proposed ratio type estimator by imposing coefficient of variation, C_x on work of Singh H. P., Tailor R., and Kakran M. S. (2004), the propose estimator it as:

$$\hat{\bar{Y}}_5 = \bar{y}\left(\frac{\bar{X}C_x+\beta_2}{\bar{x}\,C_x+\beta_2}\right) \qquad (10)$$

$$\text{Bias}(\hat{\bar{Y}}_5) = \theta\bar{Y}\left(\delta_5^2 C_x^2 - 2\rho C_x C_y\right) \qquad (11)$$

$$\text{MSE}(\hat{\bar{Y}}_5) = \theta\bar{Y}^2\left(C_y^2 + \delta_5^2 C_x^2 + -2\rho C_x C_y\right) \qquad (12)$$

where it's constant is given as: $\delta_5 = \frac{\bar{X}C_x}{\bar{X}+\beta_2}$ (13)

Singh H. P., Tailor R. (2003) proposed ratio type estimator by replacing correlation of an auxiliary variable on work of Sisodia B. V. S. and Dwivedi V.K (1981)

$$\hat{\bar{Y}}_2 = \bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right) \qquad (14)$$

$$\text{Bias}(\hat{\bar{Y}}_2) = \theta\bar{Y}\left(\delta_2^2 C_x^2 - 2\rho C_x C_y\right) \qquad (15)$$

$$\text{MSE}(\hat{\hat{Y}}_2) = \theta\bar{Y}^2\left(C_y^2 + \delta_2^2 C_x^2 + -2\rho C_x C_y\right), \quad (16)$$

where it's constant is given as: $\delta_2 = \frac{\bar{X}}{\bar{X}+\rho}$ (17)

Singh H. P., Tailor R., and Kakran M. S. (2004) proposed ratio type estimator by replacing coefficient of variation (C_x) in work of Sisodia B. V. S. and Dwivedi V.K (1981) with coefficient of kurtosis (β_2) of auxiliary variable as:

$$\hat{\hat{Y}}_6 = \bar{y}\left(\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right) \quad (18)$$

$$\text{Bias}(\hat{\hat{Y}}_6) = \theta\bar{Y}\left(\delta_6^2 C_x^2 - 2\rho C_x C_y\right) \quad (19)$$

$$\text{MSE}(\hat{\hat{Y}}_6) = \theta\bar{Y}^2\left(C_y^2 + \delta_6^2 C_x^2 + -2\rho C_x C_y\right) \quad (20)$$

where it's constant is given as: $\delta_6 = \frac{\bar{X}}{\bar{X}+\beta_2}$ (21)

Subramani J., Kumarpandiyan G.(2013) proposed an estimator by replacing the correlation (ρ) on the work of Singh H. P., Tailor R. (2003), and coefficient of kurtosis (β_2)on the work of Upadhyaya L. N., Singlh H. P, (1999) by median of auxiliary variable.

$$\hat{\hat{Y}}_3 = \bar{y}\left(\frac{\bar{X}+M_d}{\bar{x}+M_d}\right) \quad (22)$$

$$\text{Bias}(\hat{\hat{Y}}_3) = \theta\bar{Y}\left(\delta_3^2 C_x^2 - 2\rho C_x C_y\right) \quad (23)$$

$$\text{MSE}(\hat{\hat{Y}}_3) = \theta\bar{Y}^2\left(C_y^2 + \delta_3^2 C_x^2 + -2\rho C_x C_y\right), \quad (24)$$

where it's constant is given as: $\delta_3 = \frac{\bar{X}}{\bar{X}+M_d}$ (25)

Jerajuddin and Kishun (2016) proposed a ratio type estimator that minimized an error term and biasness compare to other ratio type. They are the first to use the sample size n in order to increase the precision of Cochran W.G (1940) estimator. The estimator it as:

$$\hat{\hat{Y}}_4 = \bar{y}\left(\frac{\bar{X}C_x+n}{\bar{x}\,C_x+n}\right) \quad (26)$$

$$\text{Bias}(\hat{\hat{Y}}_4) = \theta\,\bar{Y}\left(\delta_{JKP}^2 C_x^2 - 2\rho C_x C_y\right) \quad (27)$$

$$\text{MSE}(\hat{\hat{Y}}_4) = \theta\bar{Y}^2\left(C_y^2 + \delta_{JKP}^2 C_x^2 - 2\rho C_x C_y\right) \quad (28)$$

where it's constant is given as: $\delta_4 = \frac{\bar{X}C_x}{\bar{X}+\beta_2}$ (29)

2. Researched Methodology

Suleiman and Adewara (2021), proposed an improved modified ratio estimator of population mean using information on size of the sample. The bias and mean square error up to first order of approximation were obtained. The generalized estimators of the population mean, and it's constant, bias and mean square error where is given as follows:

$$\zeta_{pi} = \bar{y}\left[\alpha_i + (1-\alpha_i)\left(\frac{\bar{X}+C_{x\,n}}{\bar{x}+C_{x\,n}}\right)\right] \quad (30)$$

Table 1: Biases and Mean Square Errors (MSE) of the existing estimators

s/n	Estimator(s) ζ_{pi}	Constant δ_{pi}	Bias	MSE
1	$\bar{y}\left[\alpha_i + (1-\alpha_i)\left(\frac{\bar{X}+C_{x\,n}}{\bar{x}+C_{x\,n}}\right)\right]$	$\frac{\bar{X}}{\bar{X}+C_{x\,n}}$	$\theta\,\bar{y}\left[-\delta_{p1}\rho C_y C_x + \delta_{p1}^2 C_x^2 + \alpha_1\delta_{p1}\rho C_y C_x - \alpha_1\delta_{p1}^2 C_x^2\right]$	$\bar{Y}^2\theta\left[C_y^2 + \delta_{p1}^2 C_x^2 - 2\delta_{p1}\rho C_y C_x - \frac{A_1^2}{B_1^2}\right]$
2	$\bar{y}\left[\alpha_i + (1-\alpha_i)\left(\frac{\bar{X}C_x+\beta_2 n}{\bar{x}C_x+\beta_2 n}\right)\right]$	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2 n}$	$\theta\bar{Y}\left[-\delta_{p2}\rho C_y C_x + \delta_{p2}^2 C_x^2 + \alpha_2\delta_{pi}\rho C_y C_x - \alpha_2\delta_{p2}^2 C_x^2\right]$	$\bar{Y}^2\theta\left[C_y^2 + \delta_{p2}^2 C_x^2 - 2\delta_{p2}\rho C_y C_x - \frac{A_2^2}{B_2^2}\right]$

3	$\bar{y}\left[\alpha_i + (1-\alpha_i)\left(\frac{\bar{X}+\rho n}{\bar{x}+\rho n}\right)\right]$	$\frac{\bar{X}}{\bar{X}+\rho n}$	$\theta\bar{Y}[-\delta_{p3}\rho C_y C_x + \delta_{p3}^2 C_x^2 + \alpha_3\delta_{p3}\rho C_y C_x - \alpha_3\delta_{p3}^2 C_x^2]$	$\bar{Y}^2\theta\left[C_y^2 + \delta_{p3}^2 C_x^2 - 2\delta_{p3}\rho C_y C_x - \frac{A_3^2}{B_3^2}\right]$
4	$\bar{y}\left[\alpha_i + (1-\alpha_i)\left(\frac{\bar{X}+\beta_2 n}{\bar{x}+\beta_2 n}\right)\right]$	$\frac{\bar{X}}{\bar{X}+\beta_2 n}$	$\theta\bar{Y}[-\delta_{p4}\rho C_y C_x + \delta_{p4}^2 C_x^2 + \alpha_4\delta_{p4}\rho C_y C_x - \alpha_4\delta_p{}^2 C_x^2]$	$\bar{Y}^2\theta\left[C_y^2 + \delta_{p4}^2 C_x^2 - 2\delta_{p4}\rho C_y C_x - \frac{A_4^2}{B_4^2}\right]$
5	$\bar{y}\left[\alpha_i + (1-\alpha_i)\left(\frac{\bar{X}+\beta_1 n}{\bar{x}+\beta_1 n}\right)\right]$	$\frac{\bar{X}}{\bar{X}+\beta_1 n}$	$\theta\bar{Y}[-\delta_{p5}\rho C_y C_x + \delta_{p5}^2 C_x^2 + \alpha_5\delta_{p5}\rho C_y C_x - \alpha_5\delta_{p5}^2 C_x^2]$	$\bar{Y}^2\theta\left[C_y^2 + \delta_{p5}^2 C_x^2 - 2\delta_{p5}\rho C_y C_x - \frac{A_5^2}{B_5^2}\right]$
6	$\bar{y}\left[\alpha_i + (1-\alpha_i)\left(\frac{\bar{X}+M_d n}{\bar{x}+M_d n}\right)\right]$	$\frac{\bar{X}}{\bar{X}+M_d n}$	$\theta\bar{Y}[-\delta_{p6}\rho C_y C_x + \delta_{p6}^2 C_x^2 + \alpha_6\delta_{p6}\rho C_y C_x - \alpha_6\delta_{p6}^2 C_x^2]$	$\bar{Y}^2\theta\left[C_y^2 + \delta_{p6}^2 C_x^2 - 2\delta_{p6}\rho C_y C_x - \frac{A_6^2}{B_6^2}\right]$
7	$\bar{y}\left[\alpha_i + (1-\alpha_i)\left(\frac{\bar{X}M_d+n}{\bar{x}M_d+n}\right)\right]$	$\frac{\bar{X}}{\bar{X}M_d n+n}$	$\theta\bar{Y}[-\delta_{p7}\rho C_y C_x + \delta_{p7}^2 C_x^2 + \alpha_7\delta_{p7}\rho C_y C_x - \alpha_7\delta_{p7}^2 C_x^2]$	$\bar{Y}^2\theta\left[C_y^2 + \delta_{p7}^2 C_x^2 - 2\delta_{p7}\rho C_y C_x - \frac{A_7^2}{B_7^2}\right]$

3. The proposed estimators;

This research work was motivated by the work of Suleiman and Adewara (2021), and propose improved modified ratio estimation of population mean using regression approach. The estimators are;

$$K_1 = \bar{y}[\alpha_1 + b\varphi(\bar{X}-\bar{x})]exp\left[\frac{\bar{X}+C_x n}{\bar{x}+C_x n}\right] \quad (31)$$

$$K_2 = \bar{y}[\alpha_2 + b\varphi(\bar{X}-\bar{x})]exp\left[\frac{\bar{X}C_x+\beta_2 n}{\bar{x}C_x+\beta_2 n}\right] \quad (32)$$

$$K_3 = \bar{y}[\alpha_3 + b\varphi(\bar{X}-\bar{x})]exp\left[\frac{\bar{X}+\rho n}{\bar{x}+\rho n}\right] \quad (33)$$

$$K_4 = \bar{y}[\alpha_4 + b\varphi(\bar{X}-\bar{x})]exp\left[\frac{\bar{X}+\beta_2 n}{\bar{x}+\beta_2 n}\right] \quad (34)$$

$$K_5 = \bar{y}[\alpha_5 + b\varphi(\bar{X}-\bar{x})]exp\left[\frac{\bar{X}+\beta_1 n}{\bar{x}+\beta_1 n}\right] \quad (35)$$

$$K_6 = \bar{y}[\alpha_6 + b\varphi(\bar{X}-\bar{x})]exp\left[\frac{\bar{X}+M_d n}{\bar{x}+M_d n}\right] \quad (36)$$

$$K_7 = \bar{y}[\alpha_7 + b\varphi(\bar{X}-\bar{x})]exp\left[\frac{\bar{X}M_d+n}{\bar{x}M_d+n}\right] \quad (37)$$

Where, $\alpha_i (i = 1,2,3,4,5,6,7)$ are suitably chosen constant to be defined such that the mean square error of the proposed estimator is minimum and b_φ is slope of the regression curve, this could be obtained by regression of both study and auxiliary variable.

To obtain the bias and Mean Square Error of the propose estimators, the following properties must be adopted.

Properties of the Proposed Estimators

$\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$ such that E(e_0) = E(e_1) = 0, E($e_0{}^2$) =θC_y^2, E($e_1{}^2$) =θC_x^2, E($e_0 e_1$) = $\theta\rho C_y C_x$, where $\theta = \frac{1-f}{n}$ (38)

Bias and MSE of K_1

Now express eq (21) in error term and substitute eq (28) into equation eq (31)

$$K_1=\bar{Y}(1+e_0)\,[\alpha+b\varphi\,(\bar{X}-\bar{X}\,(1+e_1)]exp\left[\frac{\bar{X}+C_{x\,n}}{\bar{X}\,(1+e_1)+C_{x\,n}}\right] \quad (39)$$

$$K_1=\bar{Y}(1+e_0)\,[\alpha+b\varphi\,(\bar{X}-\bar{X}\;-\bar{X}e_1)]exp\left[\frac{\bar{X}+C_{x\,n}}{\bar{X}+\bar{X}e_1+C_{x\,n}}\right] \quad (40)$$

$$K_1=\bar{Y}(1+e_0)\,[\alpha-b\varphi\bar{X}e_1]exp\left[\frac{\bar{X}+C_{x\,n}}{\bar{X}+C_{x\,n}+\bar{X}e_1}\right] \quad (41)$$

$$K_1=\bar{Y}(1+e_0)\,[\alpha-b\varphi\bar{X}e_1]exp\left[\frac{1}{1+\frac{\bar{X}}{\bar{X}+C_{x\,n}}e_1}\right] \quad (42)$$

Use inverse operation for eq (42), we have

$$K_1=\bar{Y}+\bar{Y}e_0[\alpha+b\varphi\,(\bar{X}-\bar{X}\,(1+e_1)]exp\left[\frac{1}{1+\frac{\bar{X}}{\bar{X}+C_{x\,n}}e_1}\right] \quad (43)$$

$$K_1=\bar{Y}(1+e_0)\,[\alpha-b\varphi\bar{X}e_1]exp\left[\frac{1}{1+\delta_p e_1}\right] \quad (44)$$

$$K_1=\bar{Y}(1+e_0)\,[\alpha-b\varphi\bar{X}e_1]exp\left[\left(1+\delta_p e_1\right)^{-1}\right] \quad (45)$$

Where it constant is: $\delta_p=\frac{\bar{X}}{\bar{X}+C_{x\,n}}$ (46)

$$K_1=\bar{Y}(1+e_0)\,[\alpha-b\varphi\bar{X}e_1].exp\left[\left(1+\delta_p e_1+{\delta_p}^2{e_1}^2\right)^{-1}\right] \quad (47)$$

Retaining the term up to first order approximation, we have

$$K_1=(\bar{Y}+\bar{Y}e_0)\left[\alpha+\alpha b\varphi\bar{X}e_1+\alpha{\delta_p}^2{e_1}^2-b\varphi\bar{X}e_1-b\varphi\bar{X}e_1\delta_p e_1-b\varphi\bar{X}e_1{\delta_p}^2{e_1}^2\right] \quad (48)$$

Simplify the equation and subtract $\bar{Y}$ from both side of equation, we get

$$K_1-\bar{Y}=\bar{Y}\begin{bmatrix}\alpha+\alpha b\varphi\bar{X}e_1+\alpha{\delta_p}^2{e_1}^2-b\varphi\bar{X}e_1-b\varphi\bar{X}e_1\delta_p e_1-b\varphi\bar{X}e_1{\delta_p}^2{e_1}^2+\alpha e_0+\alpha b\varphi\bar{X}e_1e_0+\\ \alpha{\delta_p}^2{e_1}^2e_0-b\varphi\bar{X}e_1e_0-b\varphi\bar{X}e_1\delta_p e_0e_1-b\varphi\bar{X}e_1e_0{\delta_p}^2{e_1}^2\end{bmatrix} \quad (49)$$

Taking the expectation of both sides of equation and substitute the values of different expectations, we have bias of K_1

$$E(K_1-\bar{Y})=\bar{Y}E\begin{bmatrix}\alpha+\alpha b\varphi\bar{X}e_1+\alpha{\delta_p}^2{e_1}^2-b\varphi\bar{X}e_1-b\varphi\bar{X}e_1\delta_p e_1-b\varphi\bar{X}e_1{\delta_p}^2{e_1}^2+\alpha e_0+\alpha b\varphi\bar{X}e_1e_0+\\ \alpha{\delta_p}^2{e_1}^2e_0-b\varphi\bar{X}e_1e_0-b\varphi\bar{X}e_1\delta_p e_0e_1-b\varphi\bar{X}e_1e_0{\delta_p}^2{e_1}^2\end{bmatrix} \quad (50)$$

$$E(K_1-\bar{Y})=\bar{Y}(\alpha+\alpha\theta{\delta_p}^2\,C_x^2-b\varphi\bar{X}\delta_p\theta C_x^2+\alpha b\varphi\bar{X}\theta\rho C_yC_x-b\varphi\bar{X}\theta\rho C_yC_x) \quad (51)$$

Re-arrange the right side of the equation (41), we have

$$E(K_1-\bar{Y})=\bar{Y}(\alpha+\alpha\theta{\delta_p}^2\,C_x^2-b\varphi\bar{X}\delta_p\theta C_x^2+\alpha b\varphi\bar{X}\theta\rho C_yC_x-b\varphi\bar{X}\theta\rho C_yC_x) \quad (52)$$

$$E(K_1-\bar{Y})=\bar{Y}\left[\alpha+\;\theta(\alpha{\delta_p}^2\,C_x^2+\alpha b\varphi\bar{X}\rho C_yC_x-b\varphi\bar{X}\delta_pC_x^2-b\varphi\bar{X}\rho C_yC_x)\right] \quad (53)$$

Therefore the bias:

$$Bias\ (K_1)=\bar{Y}\left[\alpha+\;\theta(\alpha{\delta_p}^2\,C_x^2+\alpha b\varphi\bar{X}\rho C_yC_x-b\varphi\bar{X}\delta_pC_x^2-b\varphi\bar{X}\rho C_yC_x)\right] \quad (54)$$

To obtain the MSE

Square both sides of eq (44) rating the term up to first order approximation, take the expectation of both sides of equation and substitute the values of different expectations, we have MSE of K_1

$$E(K_1-\bar{Y})^2=\bar{Y}^2\begin{bmatrix}\alpha+\alpha b\varphi\bar{X}e_1+\alpha{\delta_p}^2{e_1}^2-b\varphi\bar{X}e_1-b\varphi\bar{X}e_1\delta_p e_1-b\varphi\bar{X}e_1{\delta_p}^2{e_1}^2+\alpha e_0+\alpha b\varphi\bar{X}e_1e_0+\\ \alpha{\delta_p}^2{e_1}^2e_0-b\varphi\bar{X}e_1e_0-b\varphi\bar{X}e_1\delta_p e_0e_1-b\varphi\bar{X}e_1e_0{\delta_p}^2{e_1}^2\end{bmatrix}^2$$

(55)

$$E(K_1-\bar{Y})^2=MSE(K_1)=\bar{Y}^2\left[\alpha^2-\alpha\,\theta(2{\delta_p}^2\,C_x^2+\alpha b\varphi\bar{X}\delta_p\rho C_yC_x\right] \quad (56)$$

Therefore, the mean square error

$$MSE(K_1)=\bar{Y}^2\left[\alpha^2-\alpha\,\theta(2{\delta_p}^2\,C_x^2+\alpha b\varphi\bar{X}\delta_p\rho C_yC_x\right] \quad (57)$$

Which minimize o α when equation is partially differentiate with respect to α and equate to zero, we have:

$$\frac{\partial MSE(K_1)}{\partial\alpha}=\bar{Y}^2\left[\alpha^2-\alpha\,\theta(2{\delta_p}^2\,C_x^2+\alpha b\varphi\bar{X}\delta_p\rho C_yC_x\right] \quad (58)$$

$$\bar{Y}^2\left[2\alpha-\theta(2{\delta_p}^2\,C_x^2+\alpha b\varphi\bar{X}\delta_p\rho C_yC_x\right]=0 \quad (59)$$

$$(2\alpha + \alpha\theta b\varphi\bar{X}\delta_p\rho C_y C_x) = 2\theta{\delta_p}^2 C_x^2 \quad (60)$$

$$\alpha(2 + \theta b\varphi\bar{X}\delta_p\rho C_y C_x) = 2\theta{\delta_p}^2 C_x^2 \quad (61)$$

$$\alpha = \frac{2\theta{\delta_p}^2 C_x^2}{2 + \theta b\varphi\bar{X}\delta_p\rho C_y\, tC_x} = \frac{A_1}{B_1}, \quad (62)$$

Where $b\varphi = \frac{S_{xy}}{S_x^2}$ (63)

Therefor the minimum MSE of K_1 is

$$\text{MSE}_{\min}(K_1) = \bar{Y}^2\theta\left[C_y^2 + 2\frac{A_1}{B_1}\delta_p C_x^2 - \frac{{A_1}^2}{{B_1}^2}(b\varphi\bar{X}\delta_p\rho C_y C_x + 1\right] \quad (64)$$

Therefore, the bias and Mean Square Error of other proposed estimators can be obtained in the same way. The following are generalized bias and Mean Square Errors MSE of the proposed estimator given by

$$\text{Bias}(K_i) = \bar{Y}\left[\alpha_i + \theta(\alpha_i\delta_{pi}^2 C_x^2 + \alpha_i b\varphi i\bar{X}\rho C_y C_x - b\varphi i\bar{X}\delta_{pi}C_x^2 - b\varphi i\bar{X}\rho C_y C_x)\right] \quad (65)$$

$$\text{MSE}_{\min}(K_i) = \bar{Y}^2\theta\left[C_y^2 + 2\frac{A_1}{B_1}\delta_{pi}C_x^2 - \frac{{A_i}^2}{{B_i}^2}(b\varphi i\bar{X}\delta_{pi}\rho C_y C_x + 1)\right] \quad (66)$$

$$\text{Where, } \alpha_i = \frac{2\theta{\delta_{pi}}^2 C_x^2}{2 + \theta b\varphi i\bar{X}\delta_{pi}\rho C_y\, tC_x} \quad (67)$$

4. Theoretical efficiency comparison

The proposed modified ratio estimators were set to be compared theoretically with the other existing related ratio estimators of the population mean in terms of their variances and mean square error (MSE) under simple random sampling without replacement scheme and thereby establishing efficiency condition.

Efficiency Condition of K_i(*i = 1, 2, 3, 4, 5, 6, 7*) over some related existing ratio estimators.

From the MSE of proposed modified estimator K_i and equation 2, propose estimator modified estimator is better than the mean per unit estimator.

$$V(\bar{y}) - \text{MSE}_{\min}(K_i) = \bar{Y}^2\theta\left[C_y^2 + 2\frac{A_1}{B_1}\delta_{pi}C_x^2 - \frac{{A_i}^2}{{B_i}^2}(b\varphi i\bar{X}\delta_{pi}\rho C_y C_x + 1)\right] > 0 \quad (68)$$

$$\text{Or } \quad]\frac{C_y^2 + 2\frac{A_1}{B_1}\delta_{pi}C_x^2}{b\varphi i\bar{X}\delta_{pi}\rho C_y C_x + 1} > \frac{{A_i}^2}{{B_i}^2} \quad (69)$$

(i= 1, 2, 3, 4, 5, 6, 7)

When the above equation is satisfied, K_{pi} is more efficient than $\bar{y}$

From the MSE of proposed modified estimator K_i and MSE (ζ_{pi}), propose estimator modified estimators K_i is better than the modified existing ratio type estimators by Suleiman and Adewara (2021),

$$\text{MSE}(\hat{\bar{Y}}) - \text{MSE}_{\min}(K_i) =$$

$$\bar{Y}^2\theta\left[(R^2 - {\delta_{pi}}^2)\, C_y^2 - 2(R - \delta_{pi})\frac{A_1}{B_1}\delta_{pi}C_x^2 - \frac{{A_i}^2}{{B_i}^2}(b\varphi i\bar{X}\delta_{pi}\rho C_y C_x + 1)\right] > 0 \quad (70)$$

$$\text{Or } \frac{(R^2 - {\delta_{pi}}^2)\, C_y^2 - 2(R - \delta_{pi})\frac{A_1}{B_1}\delta_{pi}C_x^2}{b\varphi i\bar{X}\delta_{pi}\rho C_y C_x + 1} > \frac{{A_i}^2}{{B_i}^2} \quad (71)$$

Table 2: Biases and Mean Square Errors (MSE) of the proposed estimators

s/n	*Estimator(s)* *Kp$_i$*	*Constant* δ_{pi}	*Bias*	*MSE*
1	$\bar{y}[\alpha_1 + b\varphi\,(\bar{X} - \bar{x})]exp\left[\frac{\bar{X} + C_{x\,n}}{\bar{x} + C_{x\,n}}\right]$	$\frac{\bar{X}}{\bar{X} + C_{x\,n}}$	$\bar{Y}[\alpha_1 + \theta(\alpha_1\delta_{p1}^2 C_x^2 + \alpha_1 b_{\varphi 1}\bar{X}\rho C_y C_x - b_{\varphi 1}\bar{X}\delta_{p1}C_x^2 - b_{\varphi 1}\bar{X}\rho C_y C_x)]$	$\bar{Y}^2\theta\left[C_y^2 + 2\frac{A_1}{B_1}\delta_{p1}C_x^2 - \frac{{A_1}^2}{{B_1}^2}(b\varphi 1\bar{X}\delta_{p1}\rho C_y C_x + 1\right]$

2	$\bar{y}[\alpha_1 + b\varphi(\bar{X} - \bar{x})]exp\left(\frac{\bar{X}C_x + \beta_2 n}{\bar{x}C_x + \beta_2 n}\right)$	$\frac{\bar{X}C_x}{\bar{X}C_x + \beta_2 n}$	$\bar{Y}[\alpha_2 + \theta(\alpha_2\delta_{p2}^2 C_x^2 + \alpha_2 b_{\varphi 2}\bar{X}\rho C_y C_x - b_{\varphi 2}\bar{X}\delta_{p2}C_x^2 - b_{\varphi 2}\bar{X}\rho C_y C_x)]$	$\bar{Y}^2\theta\left[C_y^2 + 2\frac{A_2}{B_2}\delta_{p2}C_x^2 - \frac{A_2{}^2}{B_2{}^2}(b\varphi 2\bar{X}\delta_{p2}\rho C_y C_x + 1\right]$
3	$\bar{y}[\alpha_1 + b\varphi(\bar{X} - \bar{x})]exp\left(\frac{\bar{X} + \rho n}{\bar{x} + \rho n}\right)$	$\frac{\bar{X}}{\bar{X} + \rho n}$	$\bar{Y}[\alpha_3 + \theta(\alpha_3\delta_{p3}^2 C_x^2 + \alpha_3 b_{\varphi 3}\bar{X}\rho C_y C_x - b_{\varphi 3}\bar{X}\delta_{p2}C_x^2 - b_{\varphi 3}\bar{X}\rho C_y C_x)]$	$\bar{Y}^2\theta\left[C_y^2 + 2\frac{A_3}{B_3}\delta_{p3}C_x^2 - \frac{A_3{}^2}{B_3{}^2}(b\varphi 3\bar{X}\delta_{p3}\rho C_y C_x + 1\right]$
4	$\bar{y}[\alpha_1 + b\varphi(\bar{X} - \bar{x})]exp\left(\frac{\bar{X} + \beta_2 n}{\bar{x} + \beta_2 n}\right)$	$\frac{\bar{X}}{\bar{X} + \beta_2 n}$	$\bar{Y}[\alpha_4 + \theta(\alpha_4\delta_{p4}^2 C_x^2 + \alpha_4 b_{\varphi 4}\bar{X}\rho C_y C_x - b_{\varphi 4}\bar{X}\delta_{p4}C_x^2 - b_{\varphi 4}\bar{X}\rho C_y C_x)]$	$\bar{Y}^2\theta\left[C_y^2 + 2\frac{A_4}{B_4}\delta_{p4}C_x^2 - \frac{A_4{}^2}{B_4{}^2}(b\varphi 4\bar{X}\delta_{p4}\rho C_y C_x + 1\right]$
5	$\bar{y}\left[[\alpha_1 + b\varphi(\bar{X} - \bar{x})]exp\left(\frac{\bar{X} + \beta_1 n}{\bar{x} + \beta_1 n}\right)\right]$	$\frac{\bar{X}}{\bar{X} + \beta_1 n}$	$\bar{Y}[\alpha_5 + \theta(\alpha_5\delta_{p5}^2 C_x^2 + \alpha_5 b_{\varphi 5}\bar{X}\rho C_y C_x - b_{\varphi 5}\bar{X}\delta_{p2}C_x^2 - b_{\varphi 5}\bar{X}\rho C_y C_x)]$	$\bar{Y}^2\theta\left[C_y^2 + 2\frac{A_5}{B_5}\delta_{p5}C_x^2 - \frac{A_5{}^2}{B_5{}^2}(b\varphi 5\bar{X}\delta_{p5}\rho C_y C_x + 1\right]$
6	$\bar{y}\left[[\alpha_1 + b\varphi(\bar{X} - \bar{x})]exp\left(\frac{\bar{X} + M_d n}{\bar{x} + M_d n}\right)\right]$	$\frac{\bar{X}}{\bar{X} + M_d n}$	$\bar{Y}[\alpha_6 + \theta(\alpha_6\delta_{p6}^2 C_x^2 + \alpha_6 b_{\varphi 6}\bar{X}\rho C_y C_x - b_{\varphi 6}\bar{X}\delta_{p2}C_x^2 - b_{\varphi 6}\bar{X}\rho C_y C_x)]$	$\bar{Y}^2\theta\left[C_y^2 + 2\frac{A_6}{B_6}\delta_{p6}C_x^2 - \frac{A_6{}^2}{B_6{}^2}(b\varphi 6\delta_{pi}\rho C_y C_x + 1\right]$
7	$\bar{y}\left[[\alpha_1 + b\varphi(\bar{X} - \bar{x})]exp\left(\frac{\bar{X}M_d + n}{\bar{x}M_d + n}\right)\right]$	$\frac{\bar{X}}{\bar{X}M_d n + n}$	$\bar{Y}[\alpha_7 + \theta(\alpha_7\delta_{p7}^2 C_x^2 + \alpha_7 b_{\varphi 7}\bar{X}\rho C_y C_x - b_{\varphi 7}\bar{X}\delta_{p7}C_x^2 - b_{\varphi 7}\bar{X}\rho C_y C_x)]$	$\bar{Y}^2\theta\left[C_y^2 + 2\frac{A_7}{B_7}\delta_{p7}C_x^2 - \frac{A_7{}^2}{B_7{}^2}(b\varphi 7\bar{X}\delta_{p7}\rho C_y C_x + 1\right]$

5. Numerical and Dataset for Empirical Study;

In this section, the performance of proposed modified ratio estimators using regression approach and the existing related ratio estimators of population mean using information on sample size (n) could be judged with dataset of above mentioned. Four natural populations from two sources. First two population: population 1 and 2 from Murthy (1967) while population 2 and 3 from Mukhopadhyay (2009).

Murthy (1967)

Population 1: Y=Output for 80 factories in a region and X = Number of workers, N = 80, n= 20, $\bar{Y}$ = 51.8264, $\bar{X}$= 11.2646, ρ = 0.9413, C_y = 0.3542, C_x= 0.7507, β_1 = 1.0500, β_2 = -0.0634, M_d = 7.5750

Population 2: Y=Output for 80 factories in a region and X = Fixed Capital, N = 80, n= 20, $\bar{Y}$ = 62.7652, $\bar{X}$= 13.6624, ρ = 0.9023, C_y = 0.3242, C_x= 0.6507, β_1 = 1.2501, β_2 = -0.0734, M_d = 7.5471

Mukhopadhyay (2009).

Population 3: Y=Output for 80 factories in a region and X = Number of workers, N = 40, n= 8, $\bar{Y}$ = 50.7858, $\bar{X}$= 2.3033, ρ = 0.8006, C_y = 0.3295, C_x= 0.8406, β_1 = 0.8799, β_2 = -0.4622, M_d = 1.2500

Population 4: Y=Output for 80 factories in a region and X = Fixed Capital, N = 40, n= 8, $\bar{Y}$ = 50.7858, $\bar{X}$= 9.4543, ρ = 0.8349, C_y = 0.3295, C_x= 0.6756, β_1 = 0.4622, β_2 = -0.0734, M_d = 7.0700

In this section, an evaluation of proposed modified ratio estimators was carried out and compare with the existing estimators proposed by Suleiman and Adewara (2021) mentioned above in table 2, using above populations data set, and we applied the proposed estimators and existing estimators to dataset , in order to investigate the efficiency of both estimators.

Numerical values of biases and mean square error as well as percentage relative efficiency (PRE) of the newly proposed modified ratio estimators over other existing related ratio estimators of population mean for the four population are shown in table 3 to 6

It can be observed that some proposed modified ratio estimators were having lower biases compared with other existing related estimators, while the mean square error of newly proposed modified ratio estimators were also lower as compared to other existing related estimators

Table 3: Biases, Mean Square Error and Percentage Relative Efficiency of the newly proposed modified ratio estimators and other related existing estimators using population 1 data set

Estimator	Constant	Bias	MSE	PRE
$\bar{y}$	0.000000	0.000000	12.0024	NA
$\hat{\bar{Y}}_r$	0.000000	0.608819	18.9793	66.589
ζ_{p1}	0.428661	-0.007525	1.439881	877.615
ζ_{p2}	1.176397	0.3562010	1.439996	877.545
ζ_{p3}	0.374356	-0.033941	1.439885	877.612
ζ_{p4}	1.126843	0.332096	1.439992	877.547
ζ_{p5}	0.349132	-0.046211	1.439985	877.552
ζ_{p6}	0.069208	-0.182376	1.439981	877.554
ζ_{p7}	0.810119	0.178302	1.439988	877.550
K_1	0.013264	-0.00453	0.18457	906.823
K_2	0.023440	-0.02562	1.36934	906.553
K_3	0.015643	-0.01897	1.40645	906.551
K_4	0.003245	0.00123	0.73670	906.556
K_5	0.007432	0.14535	0.34325	906.574
K_6	0.009675	-0.11672	1.52100	906.565
K_7	0.231678	0.00976	1.0045	906.556

Table 4: Biases, Mean Square Error and Percentage Relative Efficiency of the newly proposed modified ratio estimators and other related existing estimators using population 2 data set

Estimator	Constant	Bias	MSE	PRE
$\bar{y}$	0.00000	0.000000	12.63661	NA
$\hat{\bar{Y}}_r$	0.0000	1.151032	41.32765	30.577
ζ_{p1}	0.130666	-0.12673	2.056073	614.345
ζ_{p2}	0.162347	-0.107145	2..056889	614.355
ζ_{p3}	0.134805	-0.123600	2.056895	614.354
ζ_{p4}	0.169667	-0.1027724	2.056881	614.358
ζ_{p5}	0.098786	-0.1451187	2.056933	614.342
ζ_{p6}	0.087864	-0.1516441	2.056947	614.359
ζ_{p7}	0.174234	-0.10004	2.056875	614.359
K_1	0.16823	-0.03436	1.9766	668.355
K_2	0.63634	-0.10693	2.0026	668.355
K_3	0.26373	-0.07448	2.0025	668.355
K_4	0.71927	-0.08385	1.9389	668.355
K_5	0.82923	-0.01262	1.9026	668.355
K_6	0.53883	-0.01276	2.0382	668.355
K_7	0.72527	-0.00267	2.0273	668.355

Table 5: Biases, Mean Square Error and Percentage Relative Efficiency of the newly proposed modified ratio estimators and other related existing estimators using population 3 data set

Estimator	Constant	Bias	MSE	PRE
$\bar{y}$	0.0000	0.00000	28.0024	NA
$\hat{\bar{Y}}_r$	0.0000	2.46240	95.86411	29.211
ζ_{p1}	0.2551	-0.0661	10.053897	278.523
ζ_{p2}	-0.8277	-1.2856	10.053953	278.521
ζ_{p3}	0.2645	-0.5554	10.053889	278.523
ζ_{p4}	-1.1688	-1.6688	10.053966	278.521
ζ_{p5}	0.2281	-0.0964	10.053941	278.522
ζ_{p6}	0.1872	-0.1425	10.053976	278.521
ζ_{p7}	0.2646	-0.0553	10.053876	278.523
K_1	0.2343	-0.0234	7.0273	478.879
K_2	0.1321	-0.0675	7.8221	468.172
K_3	0.2315	-0.0321	7.8153	478.227
K_4	-1.4545	-0.0121	8.2673	478.672
K_5	0.2543	-0.0059	7.9262	478.383
K_6	0.3426	-0.3432	8.0002	478.002
K_7	-0.3425	-0.1243	7.9208	478.149

Table 6: Biases, Mean Square Error and Percentage Relative Efficiency of the newly proposed modified ratio estimators and other related existing estimators using population 4 data set

Estimator	Constant	Bias	MSE	PRE
$\bar{y}$	0.0000	0.0000	28.002	NA
$\hat{\bar{Y}}_r$	0.0000	1.3749	49.853	56.169
ζ_{p1}	0.6362	0.2162	8.4830	330.094
ζ_{p2}	2.3747	1.8571	8.4831	330.096
ζ_{p3}	0.5860	0.1687	8.4830	330.098
ζ_{p4}	1.6423	1.1658	8.4830	330.096
ζ_{p5}	0.5732	0.1567	8.4830	330.099
ζ_{p6}	0.1432	-0.2491	8.4300	330.100
ζ_{p7}	0.8931	0.4586	8.4530	330.097
K_1	0.4982	0.0378	4.9822	420.748
K_2	0.5873	0.0294	6.7839	420.972
K_3	0.6298	0.0098	4.8296	420.873
K_4	0.8948	0.5092	5.5826	420.425
K_5	0.6383	0.1839	8.0983	420.364
K_6	0.9830	0.3219	7.2883	420.527
K_7	0.8601	0.1930	7.0098	420.827

Observation from Table 3,

For population 1, it was observed that all newly proposed improved modified ratio estimators has lower bias and mean square error (MSE) compared to other existing modified ratio estimators considered in this study. The new proposed estimators (K_1, K_2, K_3, K_4, K_5, K_6, K_7) has bias as follow (-0.00453, -0.02562, -0.01897, 0.00123, 0.14535, 0.14535, -0.11672, 0.00976) respectively and MSE as follow (0.18457, 1.36934, 1.40645, 0.73670, 0.34325, 1.52100, 1.0045) respectively. And all the newly proposed modified ratio estimators has higher percentage relative efficiency (PRE) as (906.823) compare to percentage relative efficiency (PRE) of other existing modified ratio estimators considered in this study.

Observation from Table 4,

For population 2, it was observed that all newly proposed improved modified ratio estimators has lower bias and mean square error (MSE) compared to other existing modified ratio estimators considered in this study. The new proposed estimators ($K_1, K_2, K_3, K_4, K_5, K_6, K_7$) has bias as follow (-0.03436, -0.10693, -0.07448, -0.08385, -0.01262, -0.01276, -0.00267) respectively and MSE as follow (1.9766, 2.0026, 2.0025, 1.9389, 1.9026, 2.0382, 2.0273) respectively. And all the newly proposed modified ratio estimators has higher percentage relative efficiency (PRE) (668.355) compare to percentage relative efficiency (PRE) of other existing modified ratio estimators considered in this study.

Observation from Table 5,
For population 3, it was observed that all newly proposed improved modified ratio estimators has lower bias and mean square error (MSE) compared to other existing modified ratio estimators considered in this study. The new proposed estimators ($K_1, K_2, K_3, K_4, K_5, K_6, K_7$) has bias as follow (-0.0234, -0.0675, -0.0321, -0.0121, -0.0059, -0.3432, -0.1243) respectively and MSE as follow (7.0273, 7.8221, 7.8153, 8.2673, 7.9262, 8.0002, 7.9208) respectively. And all the newly proposed modified ratio estimators has higher percentage relative efficiency (PRE) as (478.879) compare to percentage relative efficiency (PRE) of other existing modified ratio estimators considered in this study.

Observation from Table 6,
For population 4, it was observed that all newly proposed improved modified ratio estimators has lower bias and mean square error (MSE) compared to other existing modified ratio estimators considered in this study. The new proposed estimators ($K_1, K_2, K_3, K_4, K_5, K_6, K_7$) has bias as follow (0.0378, 0.0294, 0.0098, 0.5092, 0.1839, 0.3219, 0.1930) respectively and MSE as follow (4.9822, 6.7839, 4.8296, 5.5826, 8.0983, 7.2883, 7.0098) respectively. And all the newly proposed modified ratio estimators has higher percentage relative efficiency (PRE) as (420.527) compare to percentage relative efficiency (PRE) of other existing modified ratio estimators considered in this study.

6. Conclusion

The observation from the four natural population dataset using as an empirical study for this work can be used to conclude that the newly proposed modified ratio estimators with auxiliary variable using regression approach exhibited highly relative efficiency over existing relative ratio estimators with auxiliary variable using information on sample size. In conclusion, the newly improved modified ratio estimators are new version of Suleiman and Adewara (2021) improved modified ratio estimation of population mean using information on size of sample size. Base on empirical finding, the newly improved modified estimators are recommended for estimating finite population mean of any variable of interest.

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